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APPLICATION OF THE AMBIGUITY FUNCTION TO ACOUSTIC SIGNATURES

11 1967 12 26p.

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SUMMARY

The author investigates the degradation in system performance as defined by the ambiguity function for two kinds of signal distortion common to underwater acoustic systems. For deterministic distortion the effect on signal processing is noted for distortion arising from the dispersion of the water as well as from the array. Random distortion effects caused by the inhomogeneity of the medium are defined in terms of a statistical measure on the ambiguity function.

LIST OF SYMBOLS

I_{mn} = complex shading of the (m,n) hydrophone

τ_x = $\cos(\alpha_x) - \cos(\alpha_{xR})$

τ_y = $\cos(\alpha_y) - \cos(\alpha_{yR})$

$\cos(\alpha_x)$, $\cos(\alpha_y)$ = direction cosines of radius observation vector

$\cos(\alpha_{xR})$, $\cos(\alpha_{yR})$ = direction cosines of maximum response axis

k = propagation constant

c = velocity of propagation

$\hat{a}_1$ = $a_{1x} + a_{1y}$

$\hat{a}_2$ = $a_{2x} + a_{2y} + 2\, a_{1x}\, a_{1y}$

a_{1x}, a_{1y} = expansion coefficients for $\left(\frac{1}{c}\pi M d \tau_x f\right)$ and $\left(\frac{1}{c}\pi N d \tau_y f\right)$ from Figure 2

B = $\dfrac{\cos \alpha_x}{\tau_x}$

S = salinity in parts per thousand

A = 2.34×10^{-6}

f_T (KHZ) = $21.9 \times 10^{\left[6 - 1520/(T + 273)\right]}$

T = temperature in ^{o}C

f (KHZ) = frequency of acoustic wave

B' = 3.38×10^{-6}

P = pressure in kg/cm^2 (atmospheres)

$\overline{\mu^2}$ = mean square fluctuation of the index of refraction over the path (5×10^{-9} from Liebermann)

a = 60 cm

L = length of ray travel

ΔL = the lateral displacement of the two observers (receivers)

ι = transverse displacement of the two observers

$$\mathrm{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}$$

$\mathrm{Re}\left\{\cdot\right\}$ = real part of quantity in brackets

$X(\tau,\phi)$ = classical ambiguity function

I. INTRODUCTION

In underwater acoustic systems, signal distortion is often one of the most serious factors limiting system performance. Using the ambiguity function as a measure of system performance, the following paper describes the changes that result in this measure as signal distortion is included. The sources of distortion considered are: (a) the sonifying and receiving arrays, (b) the dispersion of the water, and (c) the effect of the inhomogeneity of the water due to the small microfluctuations in temperature.

Since both deterministic and random kinds of distortion are included, two approaches to describe the effect on system performance are considered. For deterministic distortion, types (a) and (b) above, the distortion is described in terms of an equivalent transfer function. By relating this transfer function to the classical two-dimensional ambiguity function the effect on system performance is derived. For the random distortion case, type (c), the measure of system performance used is the expected value of the magnitude squared of a random ambiguity function.

II. DETERMINISTIC DISTORTION

Two sources of deterministic distortion that an underwater acoustic signal experiences are considered in this section. The first is that due to the array; the second is that due to dispersion of the medium.

By describing the frequency dependence of the array amplitude pattern and the water dispersion in a Taylor series about the signal frequency, the effect of the equivalent transfer function on system performance is easily noted.

Array

In both underwater acoustic as well as electromagnetic systems one of the two linear beamforming beam stearing techniques is often used. The first technique utilizes true time delays for each hydrophone, the second utilizes phase shifters. To describe the frequency dependence of the Fraunhofer region pattern, a regular spaced planar array is assumed; the extension to other cases is straight forward.

Assuming the array geometry of Figure 1 it is well known that the amplitude pattern is given by:

$$\Lambda\,(f, \tau_x, \tau_y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I_{mn}\, e^{i \frac{2\pi f d}{c} (m\tau_x + n\tau_y)} \quad .$$

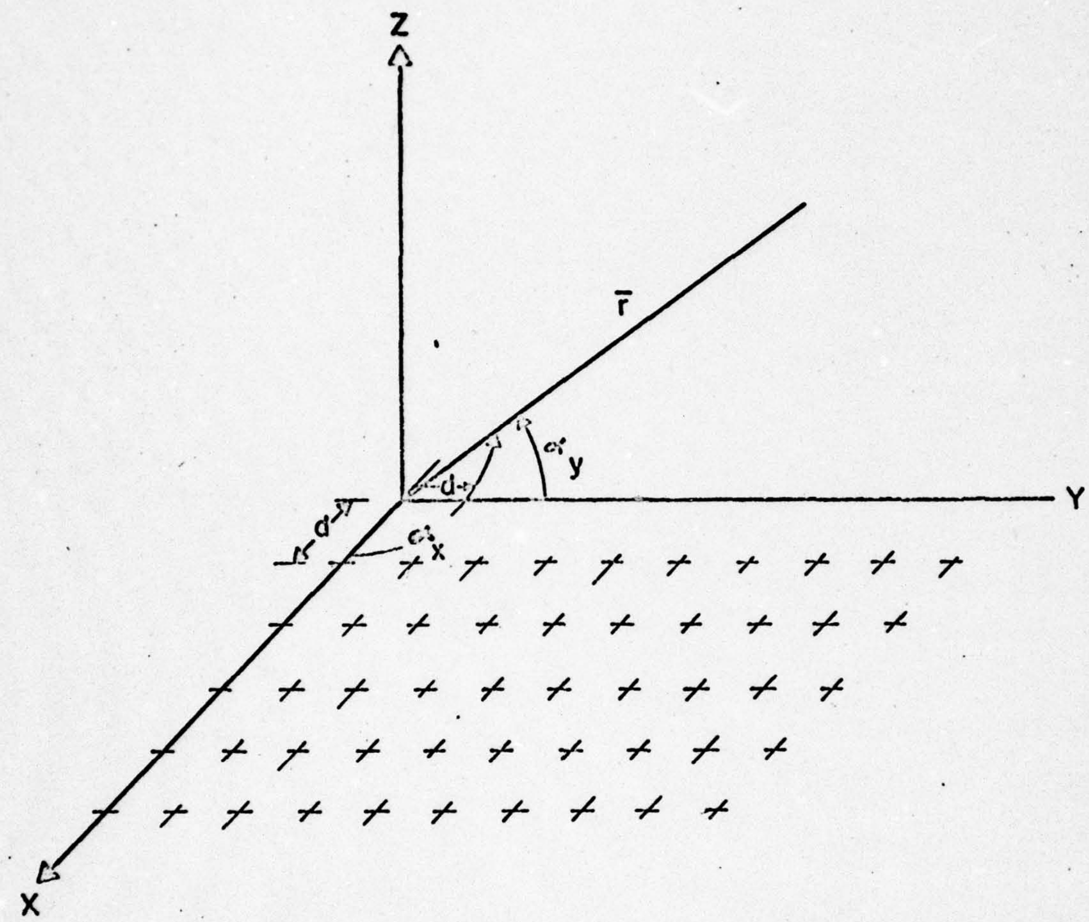

FIGURE I ARRAY DIAGRAM

From this expression a frequency dependence exists unless $\tau_x = \tau_y = o$. Therefore in general one can express this dependence as:

$$\Lambda (f, \tau_x, \tau_y) = \sum_{n=o}^{\infty} \frac{1}{n!} \frac{\partial^{(n)}}{\partial f^{(n)}} \left[\Lambda (f_o, \tau_x, \tau_y) \right] (f-f_o)^n \quad .$$

For a dense uniformly shaded linear array:

$$\Lambda (f, \tau_x) = \text{sinc} \left[\frac{1}{c} M \; d \; \tau_x \; f_o \right] \left\{ 1 + \frac{1}{f_o} \; a_1 \; (f-f_o) + \frac{1}{2f_o^{\;2}} a_2 \; (f-f_o)^2 + \ldots \right.$$

The coefficients a_1, a_2 as a function of $(\frac{1}{c} \pi \; M \; d \; \tau_x \; f_o)$ are plotted as Figure 2.

The extension to the rectangular planar array is easily shown to be:

$$\Lambda (f, \tau_x, \tau_y) = \text{sinc} \left[\frac{1}{c} M \; d \; \tau_x \; f \right] \; \text{sinc} \left[\frac{1}{c} N \; d \; \tau_y \; f \right] \quad ,$$

which can be expanded as

$$\Lambda (f, \tau_x, \tau_y) = \text{sinc} \left[\frac{1}{c} M \; d \; \tau_x \; f_o \right] \; \text{sinc} \left[\frac{1}{c} N \; d \; \tau_y \; f_o \right] \left\{ 1 + \frac{1}{f_o} \hat{a}_1 \; (f-f_o) + \frac{1}{2f_o^{\;2}} \hat{a}_2 \; (f-f_o)^2 + \ldots \right\} \quad .$$

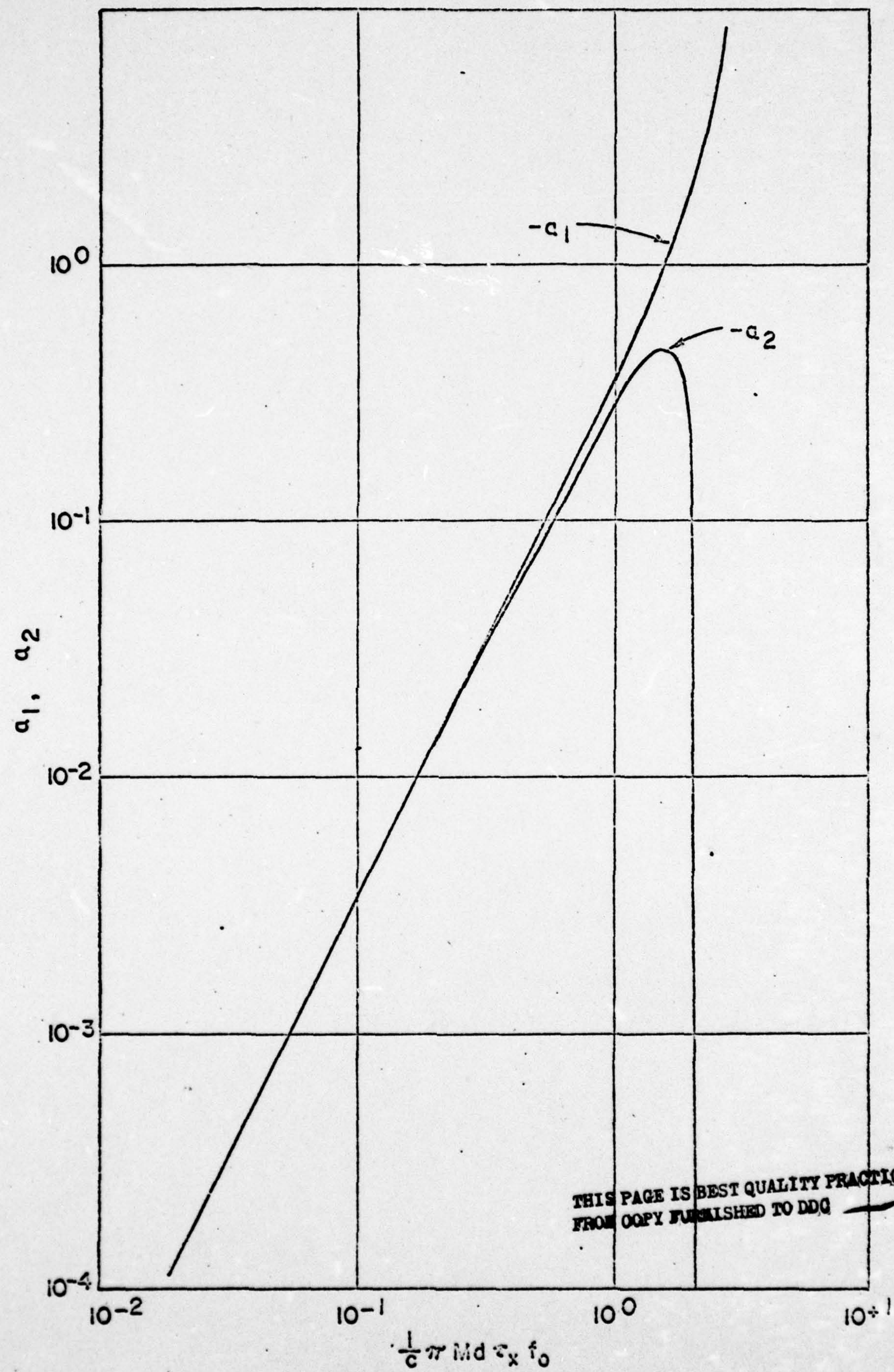

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FIGURE 2 a_1, a_2 Vs. $\frac{1}{c} \pi M d \tau_x f_0$

The usual phase shift implementation assumes a phase shift at each transducer output such that at a frequency, say f_o, a constant phase pressure wave in the desired direction is a plane. For this case

$$\Lambda\ (f, \tau_x, \tau_y) = \sum_{m=o}^{M-1} \sum_{n=o}^{N-1} I_{mn}\, e^{i \frac{2\pi d}{c}} \Big[m(f \cos \alpha_x - f_o \cos \alpha_{xR})$$

$$- n\ (f \cos \alpha_y - f_o \cos \alpha_{yR}) \Big] \quad .$$

Using the above assumptions, it easily follows that for the linear array:

$$\Lambda\ (f, \tau_x) = \operatorname{sinc} \left[\frac{1}{c} M\, d\ (f \cos \alpha_x - f_o \cos \alpha_{xR}) \right]$$

and

$$\Lambda\ (f, \tau_x) = \operatorname{sinc} \left[\frac{1}{c} M\, d\, \tau_x\, f_o \right] \left\{ 1 + \frac{B}{f_o} a_1\ (f - f_o) + \frac{1}{2} \left(\frac{B}{f_o} \right)^2 a_2\ (f - f_o)^2 + \ldots \right\} \quad .$$

The extension to the rectangular planary array yields:

$$\Lambda\ (f, \tau_x, \tau_y) = \operatorname{sinc} \left[\frac{1}{c} M\, d\, \tau_x\, f_o \right] \operatorname{sinc} \left[\frac{1}{c} N\, d\, \tau_y\, f_o \right] \left\{ 1 + \left(\frac{B}{f_o} \right) \hat{a}_1\ (f - f_o) + \frac{1}{2} \left(\frac{B}{f_o} \right)^2 \hat{a}_2\ (f - f_o)^2 \right\} \quad .$$

Therefore, it is seen that unless the maximum response axis as well as the observation vector are normal to the array ($\alpha_{xR} = \alpha_x = 0$ for the linear array and $\alpha_{xR} = \alpha_{yR} = \alpha_x = \alpha_y = 0$ for the planar array) this array function is frequency dependent. Although the above expressions become indeterminate at $\tau_x = 0$, it is possible to show that $\frac{B}{f_o} a_1 = \frac{B}{f_o} \hat{a}_1 = 0$, and

$$\left(\frac{B}{f_o}\right)^2 a_2 = -\frac{1}{3}\left[\frac{M\pi d}{c}\cos\alpha_{xR}\right]^2 ,$$

$$\left(\frac{B}{f_o}\right)^2 \hat{a}_2 = -\frac{1}{3}\left(\frac{M\pi d}{c}\cos\alpha_{xR}\right)^2 - \frac{1}{3}\left(\frac{N\pi d}{c}\cos\alpha_{yR}\right)^2 .$$

<u>Dispersion</u>

A great deal of work has been directed to the characterization of the dispersive effects of sea water on sound. It is now generally concluded that the significant factors include the relaxation effects of the $MgSO_4$ ions and viscosity. Schulkin and Marsh[1] have suggested that sound loss follows:

$$\alpha \text{ (nepers/meter)} = \left(\frac{SA\ f_T\ f_{(KHZ)}^{\ 2}}{f_T^2 + f_{(KHZ)}^{\ 2}} + \frac{B'\ f_{(KHZ)}^{\ 2}}{f_T}\right)\left(1 - 6.54 \times 10^{-4} P\right) .$$

Expanding this expression as

$$H(f) = H(f_o) \sum b_n (f - f_o)^n ,$$

the coefficients b_n as a function of f_o are shown as Figures 3, 4, and 5 for ranges of 1,000, 10,000, and 50,000 yards. These assume a temperature of 10°C, salinity of 35 parts per thousand, and zero atmospheres.

System Performance

Using the system model as shown in Figure 6, the effect on system performance of the deterministic signal distortion considered above can be obtained. By assuming the Doppler approximation*, it follows that the equivalent transfer function is given by:

$$H(f) = \Lambda_s(f-\Phi, \tau_x, \tau_y)\, H_f(f-\Phi)\, H_R(f)\, \Lambda_R(f, \tau_x, \tau_y)$$

$$= \Gamma \sum A_n (f-f_o)^n$$

From Appendix I, it is seen that the response is given by

$$R(\tau, \Phi) = 2\,\mathrm{Re}\left\{ \Gamma\, e^{i2\pi f_o \tau} \sum A_n^* \left(\frac{-1}{i2\pi}\right)^n \frac{\partial^{(n)}}{\partial \tau^{(n)}} \left[X(\tau, \Phi)\right] \right\}$$

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* For a discussion of this approximation see Remley[2].

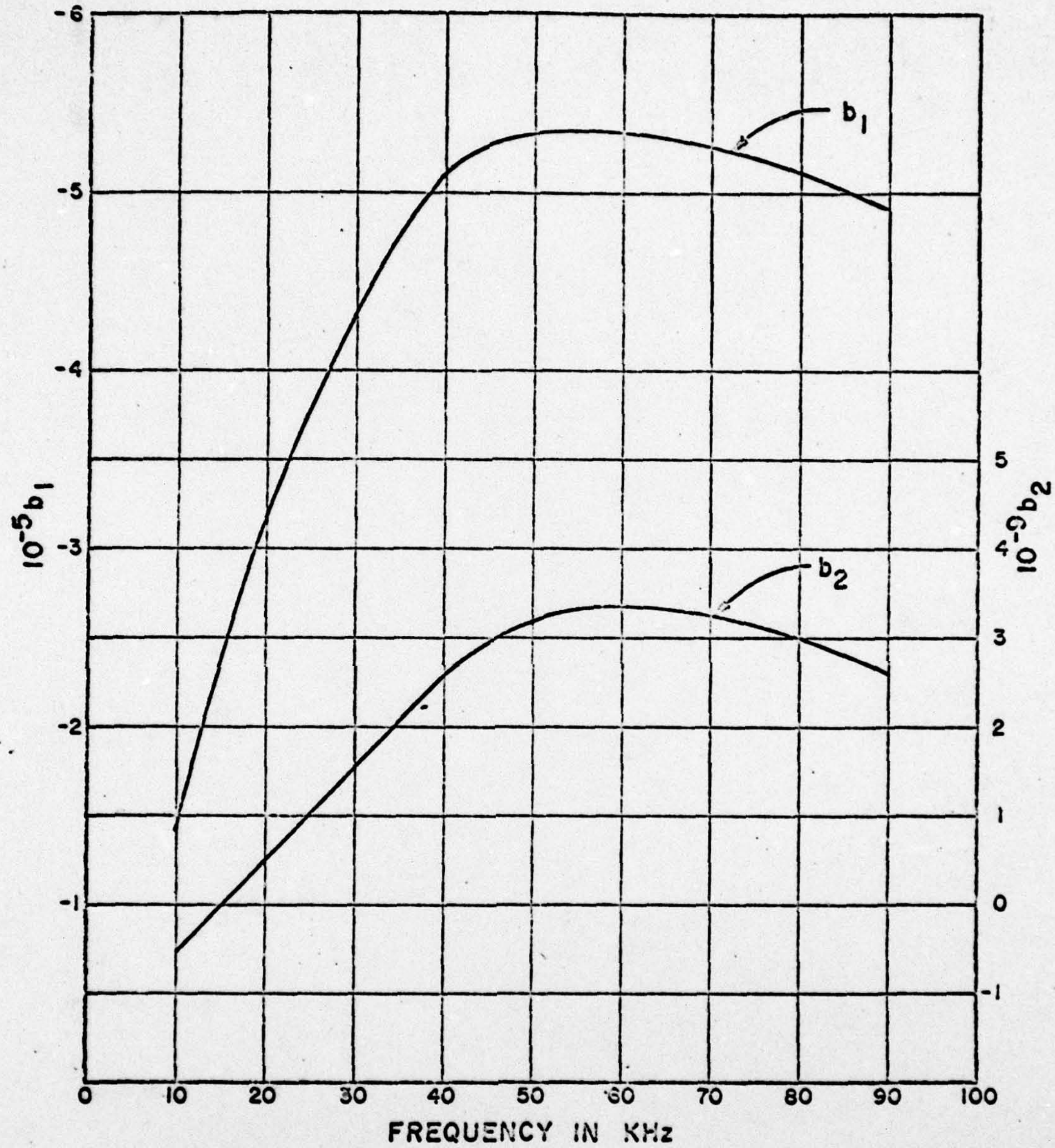

FIGURE 3 b_1, b_2 VS FREQUENCY FOR 1000 METERS

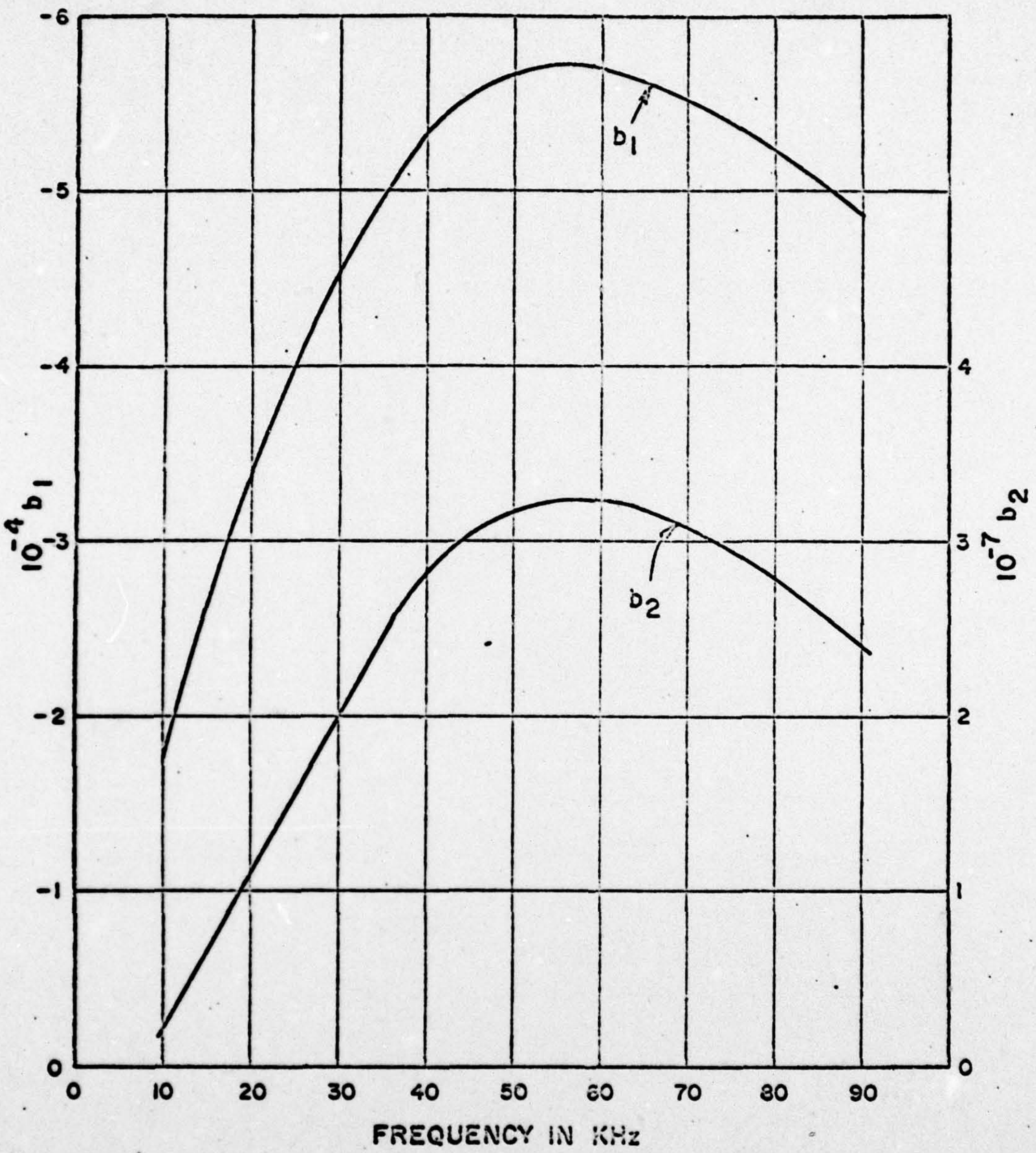

FIGURE 4 b_1, b_2 VS FREQUENCY FOR 10,000 METERS

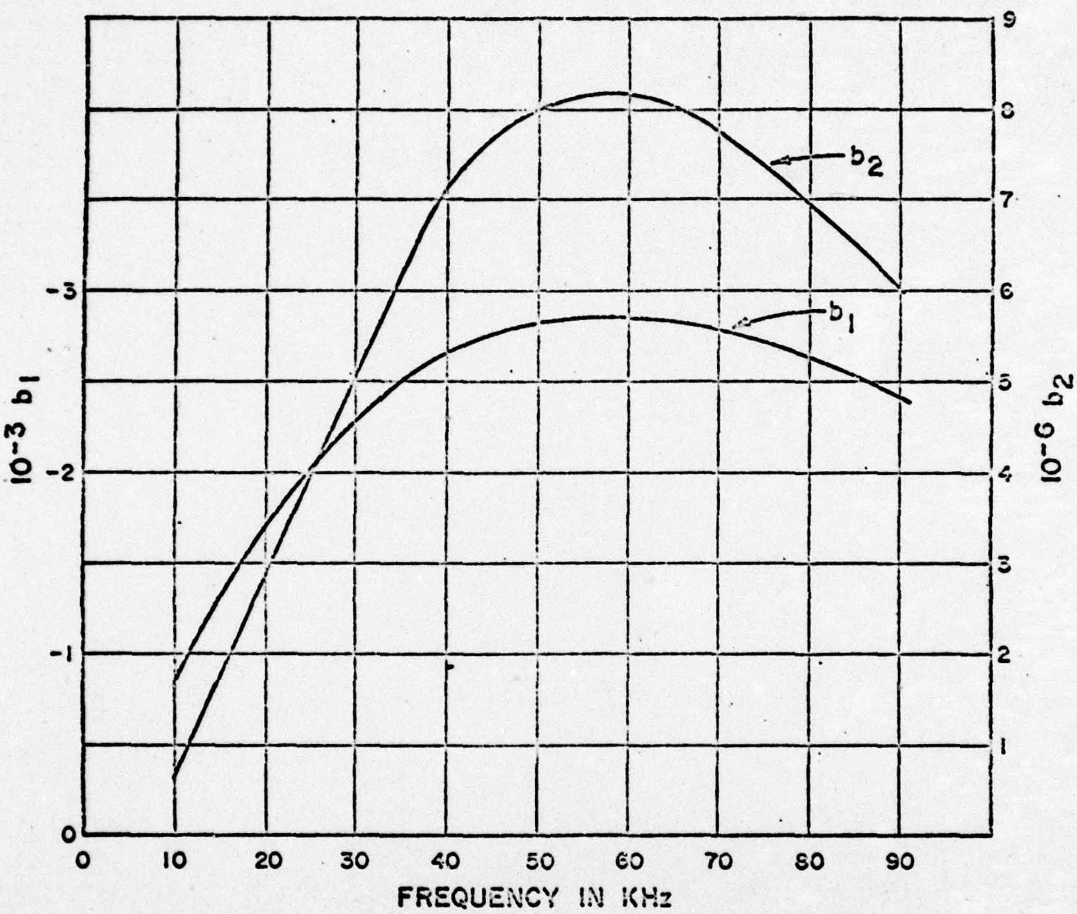

FIGURE 5 b_1, b_2 VS FREQUENCY FOR 50000 METERS

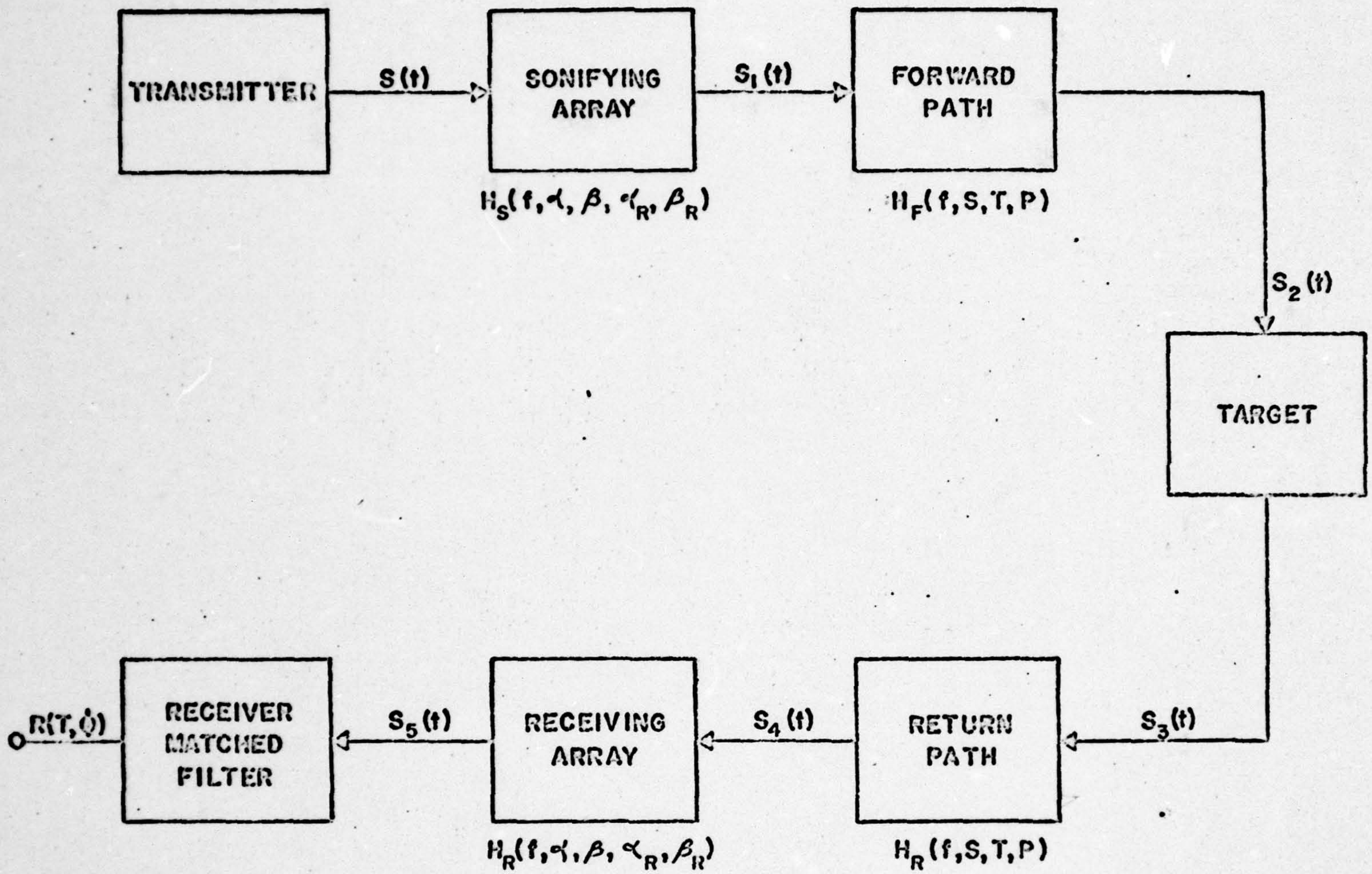

FIGURE 6 SYSTEM MODEL

III. RANDOM DISTORTION

In addition to the above sources of distortion is the distortion caused by the inhomogeneity of the water. There are small microfluctuations in temperature which give rise to random fluctuations in the velocity of sound from point to point in the ocean. If one considers rays propagating through these microfluctuations the sound will not be constant but will fluctuate about the mean from ray to ray.

Liebermann[3] has measured the fluctuations of temperature at depths of 30-60 meters using a fast acting thermometer mounted on a submarine. He found that the mean temperature fluctuation was 0.04°C and the correlation of these fluctuations as a function of distance is given by $e^{-\frac{|x|}{a}}$ where a = 60 cm.

Chernov[4] using Rytov's Method was able to derive certain covariance functions for the phase and normalized amplitude fluctuations for small random deviations in the refractive index. Of particular interest here are his derivations of the covariance functions of both amplitude and phase as a function of distance both along the ray path as well as normal to the ray path (see Figure 7).

For lateral (along the ray path) displacement of ΔL he shows, using Liebermann's results as discussed above, that the covariance for both amplitude and phase is given by:

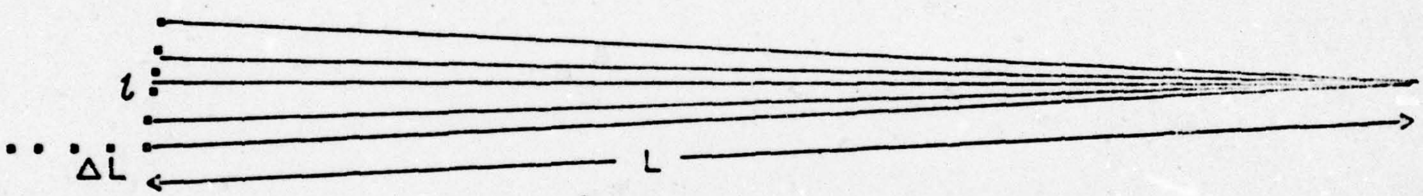

FIGURE 7 RAY PATH MODEL

$$R(\Delta L) = \frac{\frac{\sqrt{\pi}}{2}\,\overline{\mu^2}\,k^2\,a\,L}{1 + \left(\frac{2\Delta L}{ka^2}\right)^2} .$$

For the case of transverse displacements of ι (normal to the ray path) he shows that the covariance for both amplitude and phase is given by:

$$R(\iota) = \frac{\sqrt{\pi}}{2}\,\overline{\mu^2}\,k^2\,a\,L\,e^{-\frac{\iota^2}{a^2}} .$$

The correlation between amplitude and phase is given by:

$$-\rho = \frac{ka^2}{4L} \log\left(\frac{4L}{ka^2}\right) .$$

It is interesting to note that if the correlation intervals are defined as the values of ΔL and ι for which the covariance functions reach 0.5 of their maximum value we obtain

$$\Delta L = 0.5\,k\,a^2$$
$$\iota = 0.835\,a .$$

Therefore, note that decorrelation along the path can be much slower than that normal to the path.

System Performance

From the above discussion it follows that if one assumes that the ray path is changing over the signal transmission, as would occur in the case of a moving source receiver, the received signal could be represented as:

$$\left[1 + x(t)\right] \; s\left|t - y(t)\right| \quad .$$

As shown in Appendix II, if this signal is matched filter detected, the response is given by:

$$E\left\{\left|x(\tau,\phi)\right|^2\right\} = \left|x(\tau,\phi)\right|^2 * R(\phi) \quad .$$

Here, $R(\phi)$ is given by the Fourier transform of $\left[1 - R_y(o) - R_y(\tau) - R_x(\tau)\right]$. From the above, however, the expressions for $R_y(\tau) = R_x(\tau)$, which permits us to write $\left|1 - R(o) - 2\,R(\tau)\right|$.

Here, for a velocity either along the ray path of V_Δ or normal, the ray path of V_t we easily obtain for $R(\tau)$

$$R(\tau) = \frac{\sqrt{\pi}}{2} \; \frac{\overline{\mu^2}\, k^2\, aL}{1 + \left(\dfrac{2\, V_\Delta\, \tau}{k\, a^2}\right)^2}$$

$$R(\tau) = \frac{\sqrt{\pi}}{2} \; \overline{\mu^2}\, k^2\, aL\, e^{-\frac{V_t^2}{a^2}\tau^2}$$

IV. CONCLUSION

For the deterministic distortion case it is seen that the effect on the system response function can be derived in terms of coefficients of the Taylor Series expression of the distortion and derivatives of the classical ambiguity function. The use of this technique is attractive when the series converges rapidly. In addition curves were derived to define the coefficients for array derived distortion as well as anomalous absorption.

For the random distortion case it was shown that the results of the microfluctuations in temperature could be accounted for by convolving the resultant spectral density of the error signal with the classical ambiguity function.

APPENDIX I

Deterministic Distortion

It was found that the equivalent transfer function for the deterministic distortion case could be represented as:

$$H\ (f) = \Gamma \sum A_n\ (f-f_o)^n$$

It easily follows for the distortion sources considered here (this is not generally true) that:

$$H\ (f) = \Gamma \sum A_n\ (f-f_o)^n + A_n^*\ (-f-f_o)^n$$

If the transmitted narrow band signal is given by

$$S\ (f) = \alpha\ (f-f_o) + \alpha^*\ (-f-f_o)$$

and the receiver matched filter's transfer function is

$$H_n\ (f) = \left[\alpha^*\ (f-f_o + \phi) + \alpha\ (-f-f_o + \phi)\right] e^{-i2\pi fT}$$

it easily follows that the response as a function of doppler shift ϕ and time shift $\tau = T - t$ is given by:

$$R\ (\tau,\ \phi) = \Gamma\ e^{i2\pi f_o \tau} \sum_n \frac{A_n^*}{(-i2\pi)^n} \frac{\partial^{(n)}}{\partial\ \tau^{(n)}} \left[X\ (\tau,\ \phi)\right]$$

+ complex conjugate

APPENDIX II

In this section it is desired to compute the expected value of the magnitude squared of the response function $R(\tau,\phi)$ in terms of the classical ambiguity function $X(\tau,\phi)$. If we assume narrow band signals it easily follows that the time delay random variable is equivalent to a phase change $y(t) = 2\pi f_o y(t)$ which permits us to write:

$$E\left\{ \left|R(\tau,\phi)\right|^2 \right\} = E\left\{ \iint \left[1+x(t_1)\right] \left[1+x(t_2)\right] e^{i\left[y(t_1)-y(t_2)\right]} S(t_1)\ S^*(t_1+\tau)\ S^*(t_2)\ S(t_2+\tau)\ e^{-i2\pi\phi(t_1-t_2)}\ dt_1\ dt_2 \right\}$$

$$= \iint R(t_1-t_2)\ S(t_1)\ S^*(t_1+\tau)\ S^*(t_2)\ S(t_2+\tau)\ e^{-i2\pi\phi(t_1-t_2)}\ dt_1\ dt_2 .$$

Here we have

$$R(t_1-t_2) = E\left\{ \left[1+X(t_1)\right] \left[1+X(t_2)\right] e^{i\left[y(t_1)-y(t_2)\right]} \right\}$$

$$= E\left\{ \left[1+X(t_1)\right] \left[1+X(t_2)\right] \right\} E\left\{ e^{i\left[y(t_1-y(t_2)\right]} \right\}$$

Now, by assuming that the random processes are stationary Gaussian and making use of the Wiener-Kninchin theorem yields

$$E\left\{ \left|R(\tau,\phi)\right|^2 \right\} = \int \left|X(\tau,\xi)\right|^2 R(\phi-\xi)\ d\xi$$

$$= \left|X(\tau,\phi)\right|^2 * R(\phi)$$

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3. Liebermann, L. J., "Effect of Temperature Inhomogeneities in the Ocean on the Propagation of Sound," JASA, 23, 1951.

4. Chernov, L. A., "Wave Propagation in a Random Medium," (English Translation by R. A. Silvermann), McGraw-Hill Book Co., New York, N.Y.